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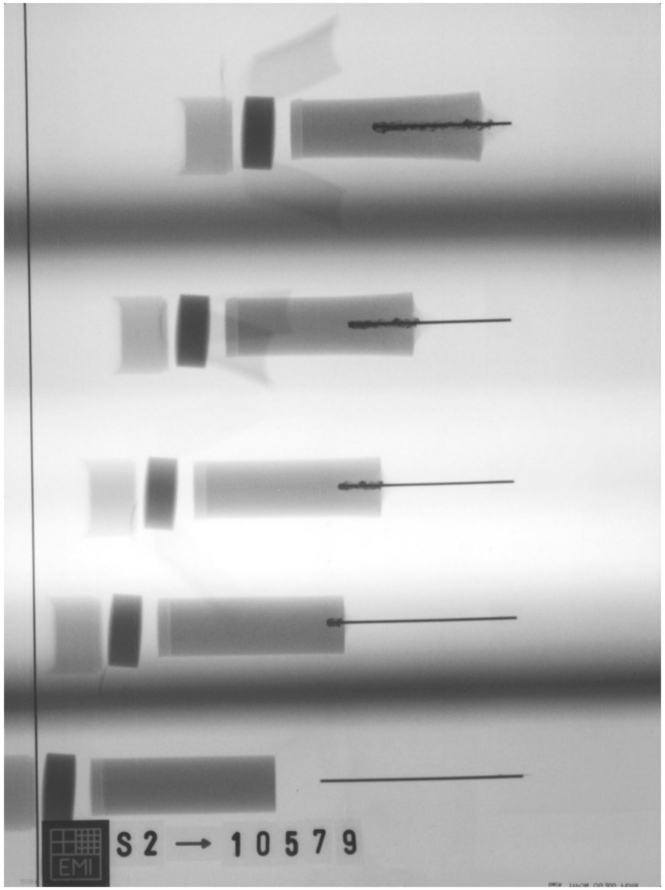
Simulations of a Gold Rod into Borosilicate Glass using Experimentally Determined Constitutive Constants

Charles E. Anderson, Jr.[1]

Katie A. McLoud[2]

[1]*CEA Consulting, San Antonio, TX* [2]*Southwest Research Institute, San Antonio, TX*



Background—1

- About 2004, began an investigation of penetration and failure of long rods into glass
- Experiments conducted at Ernst-Mach-Institut under subcontract to Southwest Research Institute (work funded by US Army TARDEC)
- Experiments were done in the reverse ballistics mode, with a 1-mm diameter gold rod suspended and then impacted by a 20-mm diameter borosilicate (Borofloat®33) glass cylinder
- Ultra-high-speed photography and flash X-rays were used to record the position of the failure front and penetration front, respectively, as a function of time

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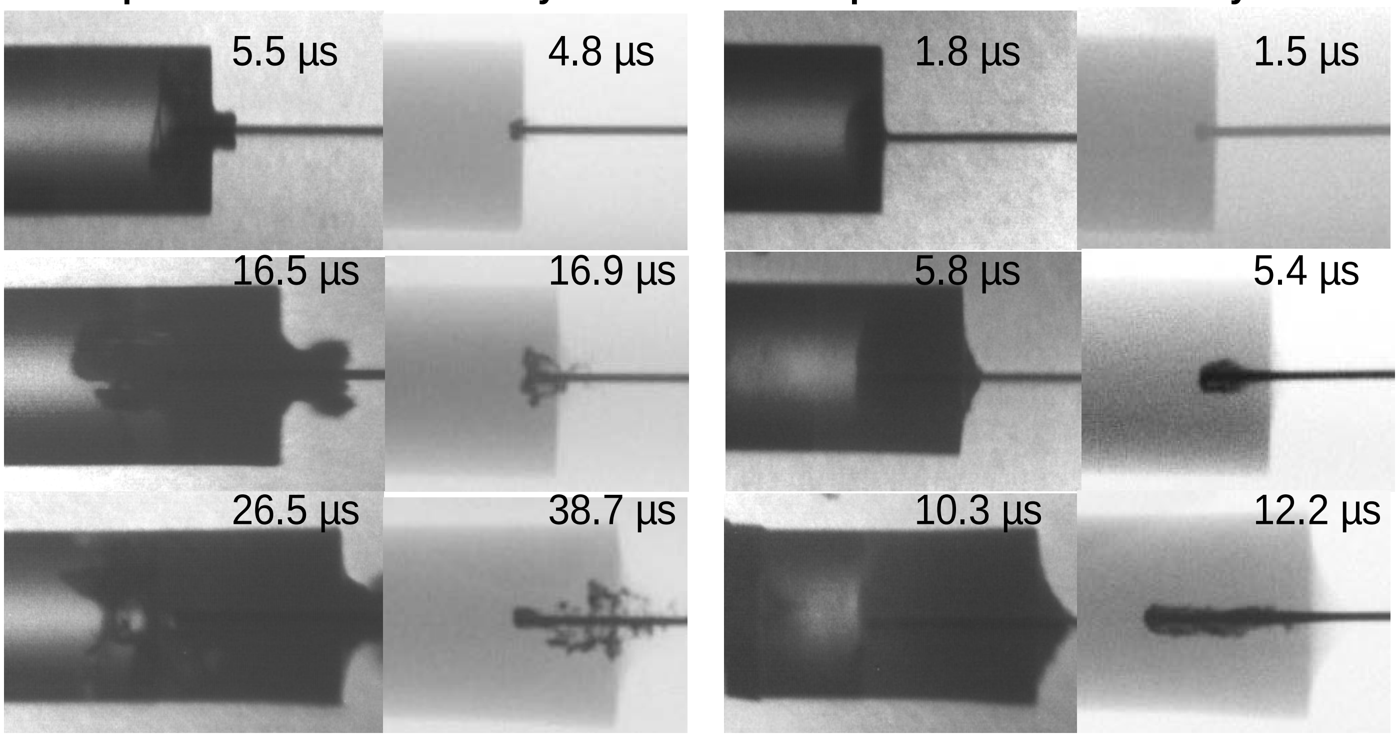
Au Rod Penetration of Borofloat Glass

optical | X-ray | optical | X-ray



Exp. 10557, v_P = 786 m/s | **Exp. 10585, v_P = 2328 m/s**

Note that failure front is outrunning the penetration front

Fraunhofer Institut Kurzzeitdynamik Ernst-Mach-Institut

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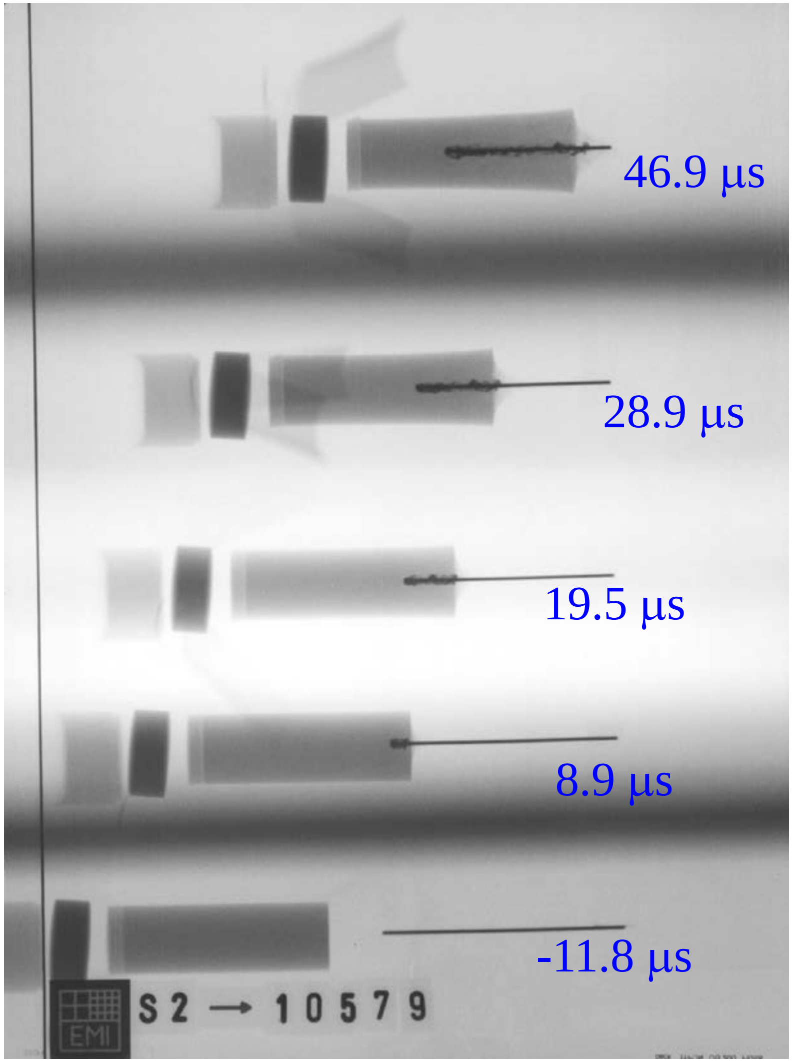
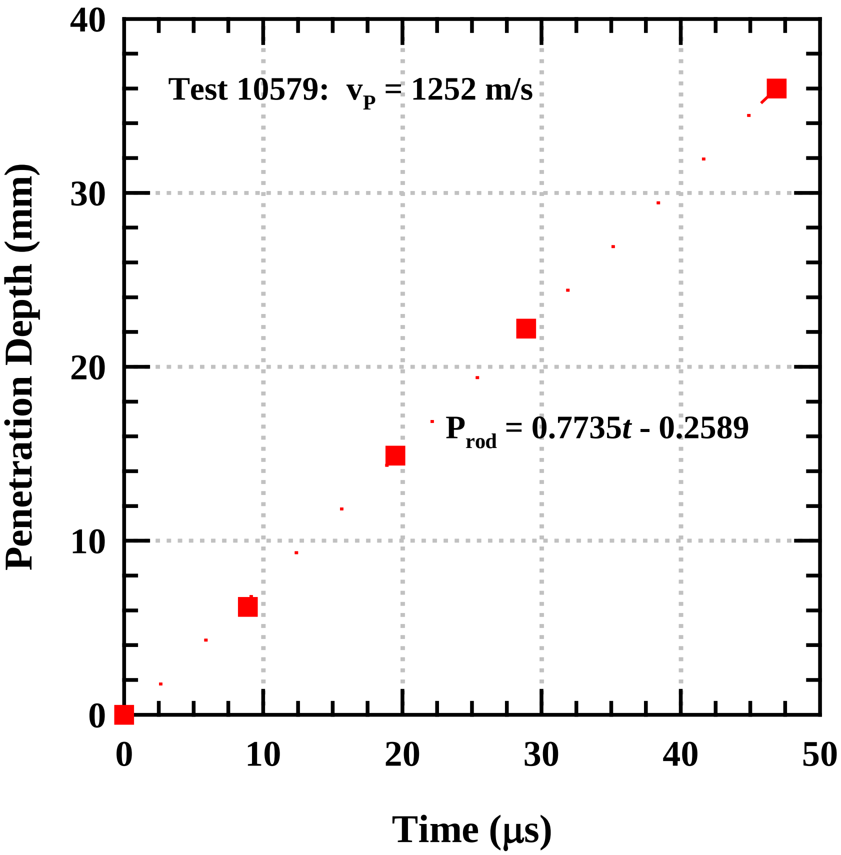
Test 10579; v_p *= 1252 m/s*

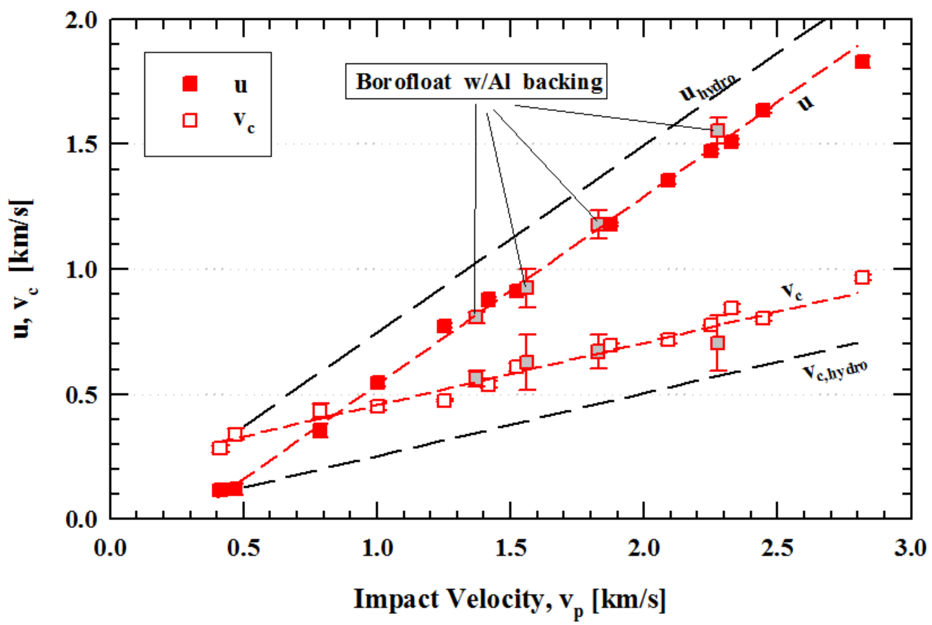




Slope is the penetration velocity

Summary of Experimental Results

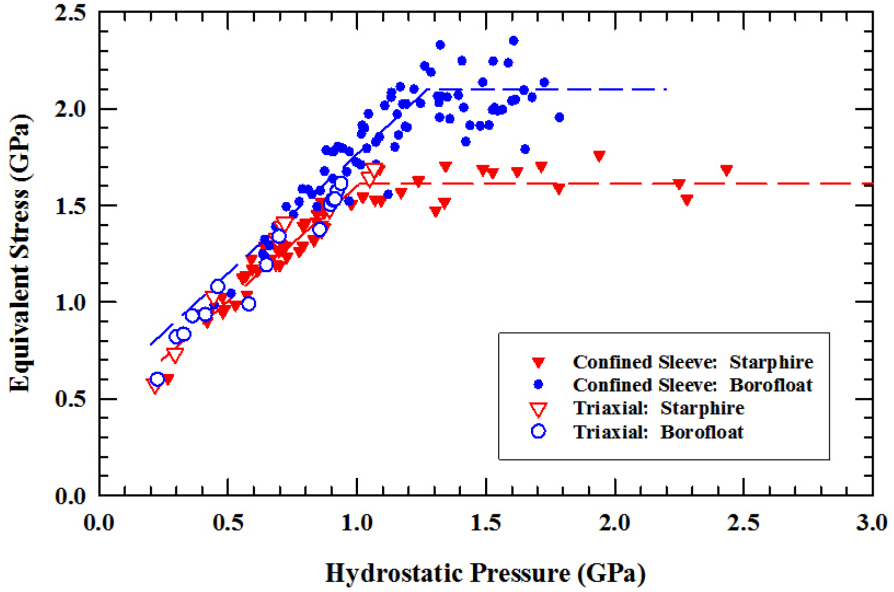
- Plot the penetration and consumption velocities as a function of the impact velocities
- Use linear regression to determine u vs. v_p and v_c vs. v_p



- $u = 0.7539\ v_p - 0.2155$
- $v_c = 0.2493\ v_p + 0.2077$
- Theory: $u + v_c = 1.0\ v_p$
- $u + v_c = 1.0032\ v_p - 0.0078$

Background—2

- In 2006, we began to conduct characterization experiments on borosilicate and soda-lime glasses to support, ultimately, development of a computational constitutive model for glass
  - Intact and damaged glass
  - Strength as a function of confinement pressure



Damaged Glass

Objective of this Work

- *Can simulations reproduce the experimental results using the results of the characterization experiments?*
- *But first, get some understanding of the uncertainty in the experimental data*

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Regression Analysis

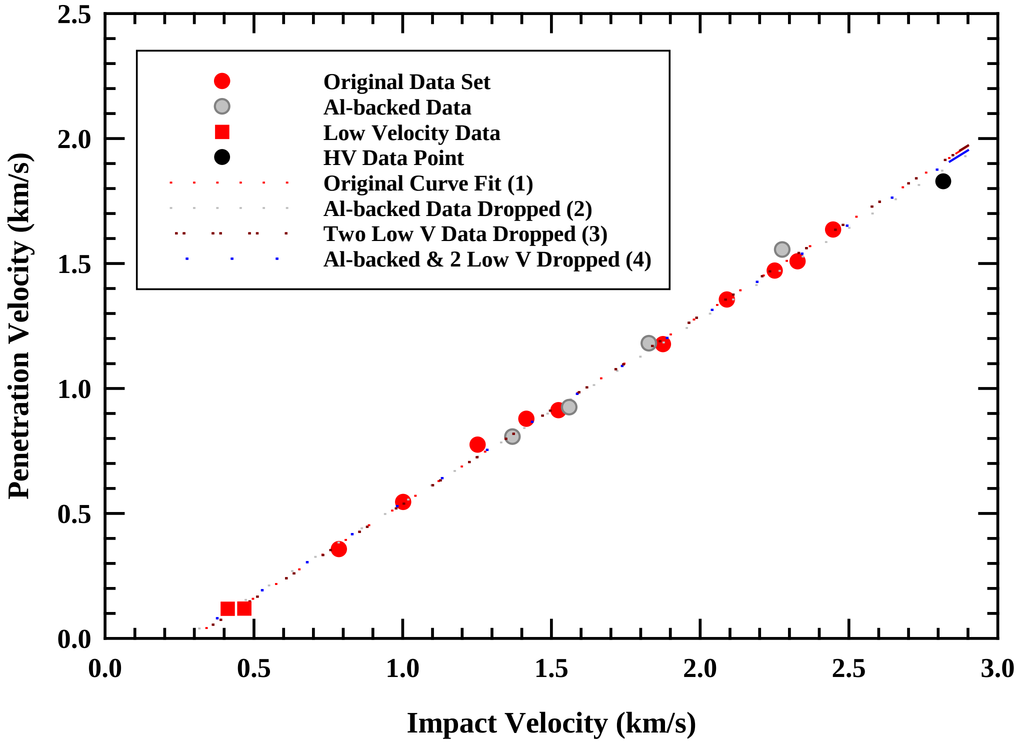
Fit No.	Regression Fit	Fit Std. Error (km/s)	Slope Std. Error	Identifying Remarks
1	$u = 0.7539v_p - 0.2155$	0.0311	0.0110	Original data set; this fit used (0,0) as a data point in *P-t* fits of experimental data
2	$u = 0.7344v_p - 0.1925$	0.0285	0.0105	Original data set but did not include Al-backed data
3	$u = 0.7559v_p - 0.2192$	0.0288	0.0153	Original data set but dropped the 2 lowest velocity data points
4	$u = 0.7424v_p - 0.1989$	0.0244	0.0139	Original data set but dropped the 2 lowest velocity data points and the Al-backed data points
5	$u = 0.7361v_p - 0.1796$	0.0304	0.0161	Data set w/o (0,0) point in *P-t* fit; 2 lowest velocity data points not included
6	$u = 0.7200v_p - 0.1530$	0.0226	0.0129	Data set w/o (0,0) point in *P-t* fit; 2 lowest velocity data points and Al-backed data not included

Get slightly different fits depending on which data to include in the analysis

The slopes of u vs. v_p change less than 5%

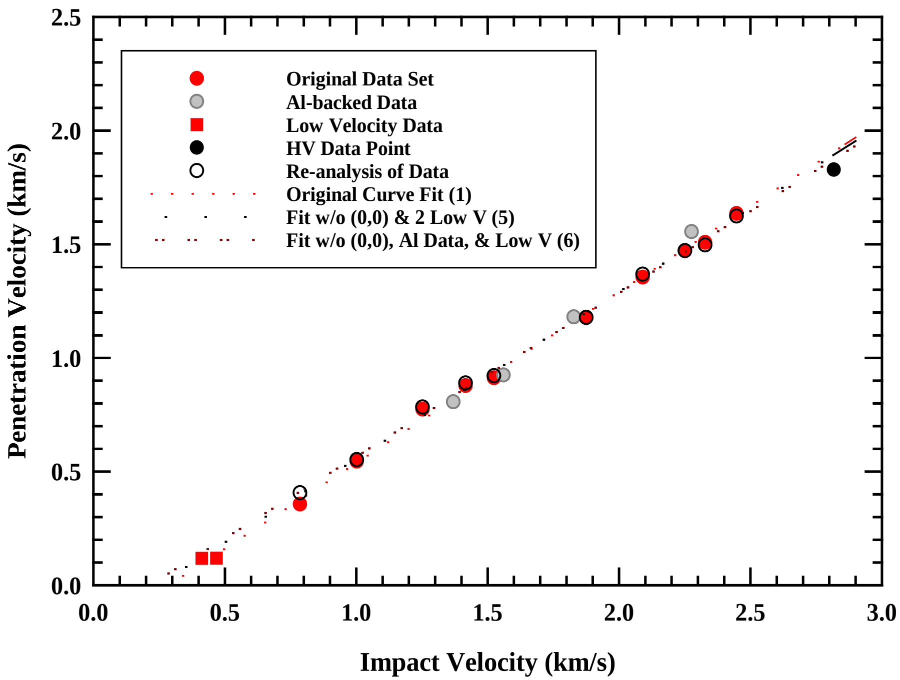
Results of Analysis of Experimental Data

- Different coefficients from the regression analysis depending upon the assumptions



Re-analysis of Experimental Data

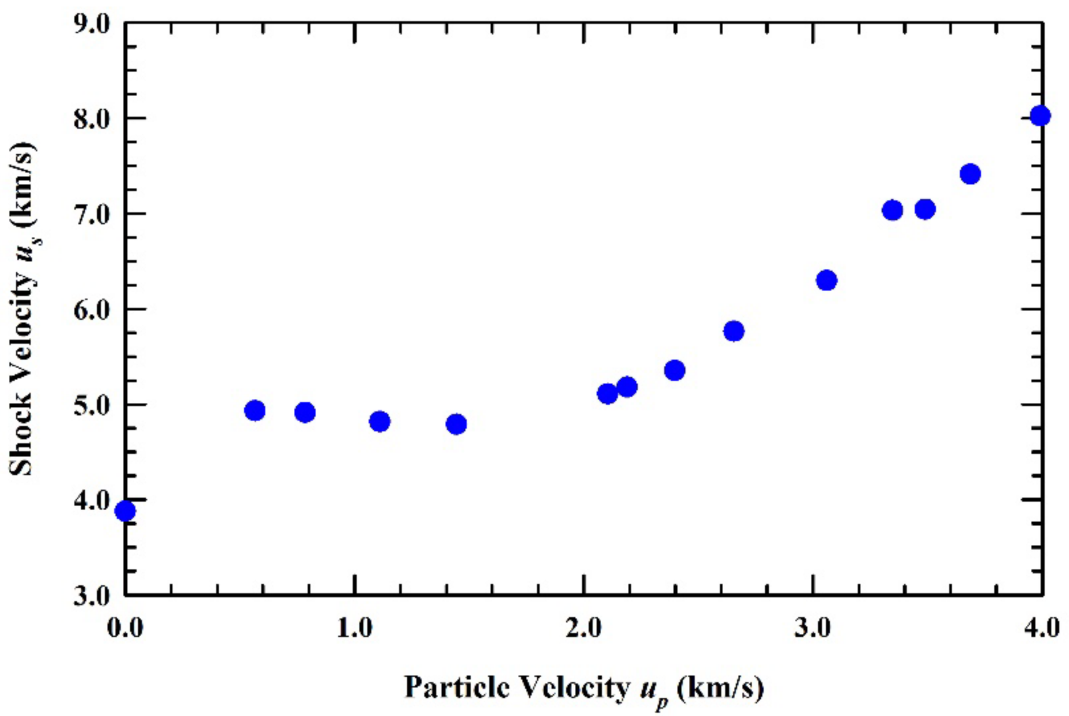
- Originally, regression analysis of P-t data included the (0,0) point (since know the time of impact)
- However, can have some dwell at early times, particularly at the lower impact velocities
- Additionally, effects of the impact shock persists for a few microseconds
- Redid regression analysis without the (0,0) point



Believe Fit No. 6 most appropriate

Equation of State
Borosilicate Glass

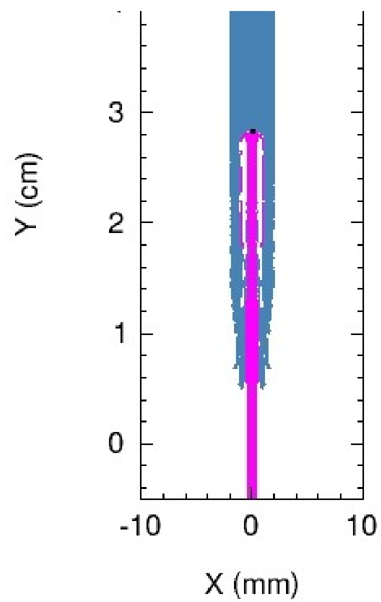
- *Glass is highly compressible*



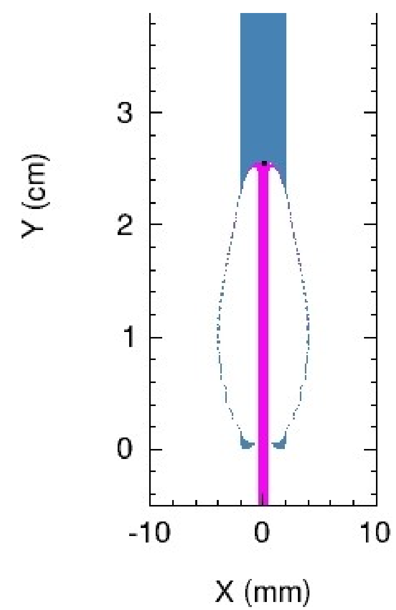
Data from Marsh, ***LASL Shock Hugoniot Data***

Hugoniot Response

- $u_s = c_o + k \bullet u_p$ $k = 0.001$



0.412 km/s



2.447 km/s

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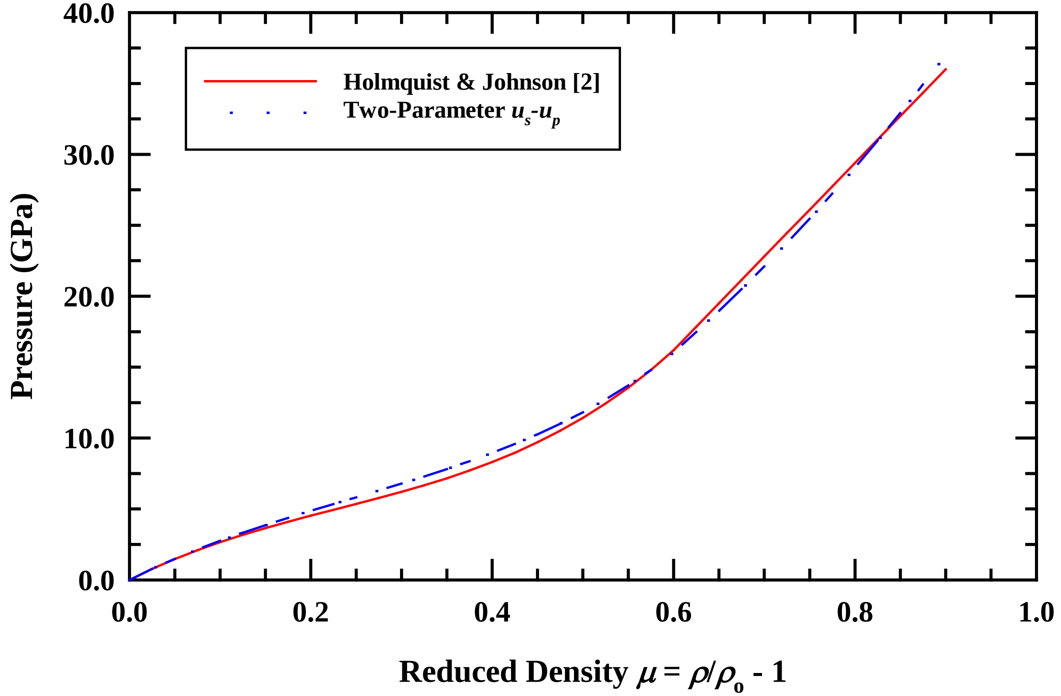
Equation of State

- $$P = K_1\mu + K_2\mu^2 + K_3\mu^3 \qquad \mu = \frac{\rho}{\rho_o} - 1$$
- $$u_s = c_o + k_1 u_p + \frac{k_2}{c_o} u_p^2$$



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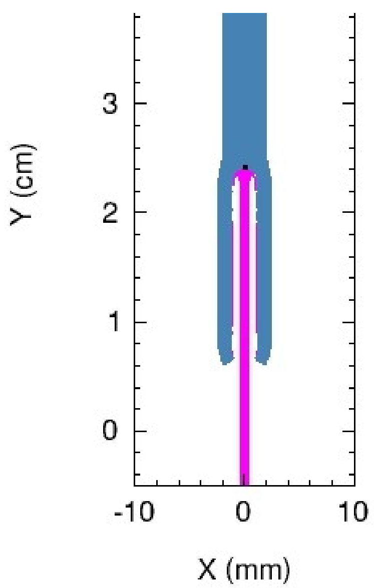
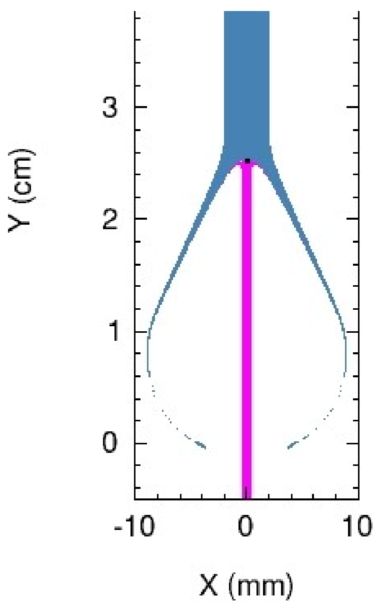
Hugoniot Response

- $$u_s = c_o + k_1 u_p + \frac{k_2}{c_o} u_p^2$$



0.412 m/s



2.447 km/s

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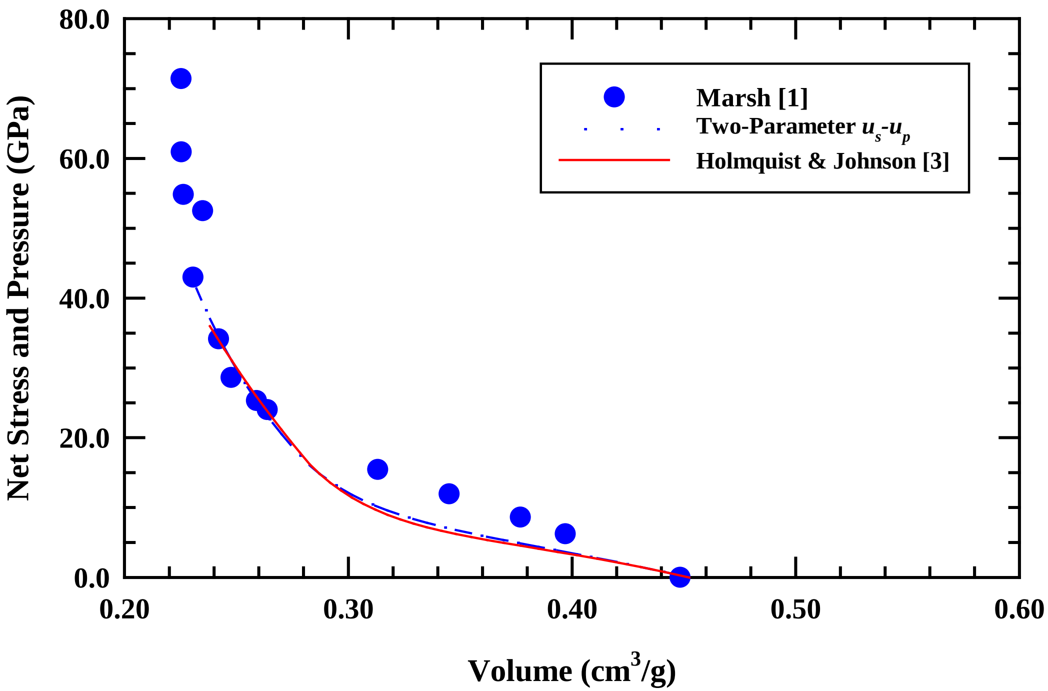
Equation of State & Hugoniot



Simulations

- *Wavecode CTH, cylindrically symmetric option*
- *Geometry*
  - *1-mm diameter rod, 70-mm long*
  - *20-mm diameter glass, 60-mm long*
- *Square zoning throughout the computational grid*
  - *0.07 mm on a side*
  - *Slightly more than 14 zones across the diameter of the rod*
- *Fully resolved numerical simulations*

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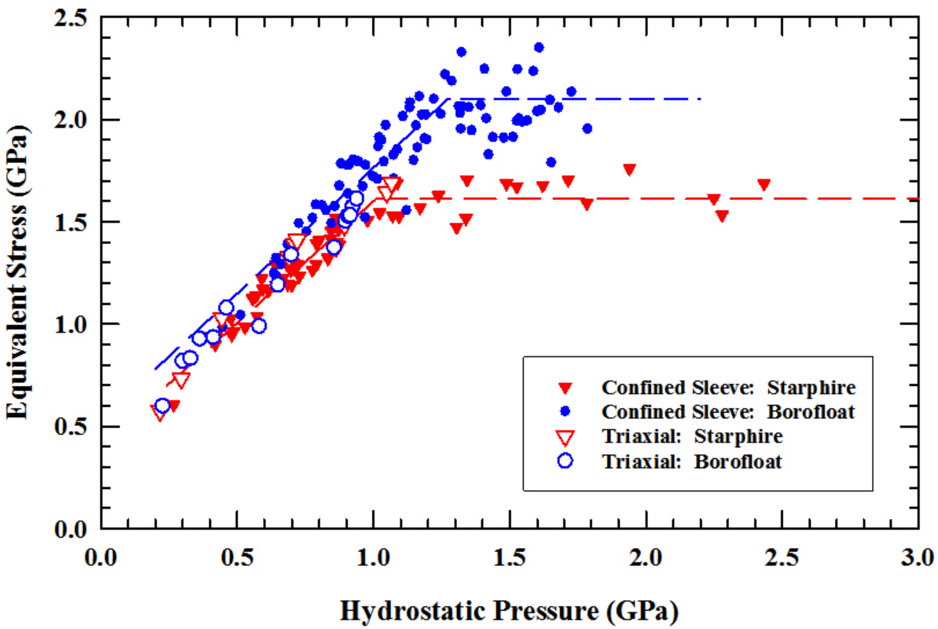
Strength Model

- *Assumption: penetrating damaged glass*
- *Constitutive model: Drucker-Prager*

$$\sigma_{eq} = \begin{cases} Y_o + \beta P & P \le P_{cap} \\ Y_{cap} & P \ge P_{cap} \end{cases}$$

$$Y_o = 0.038\,\text{GPa}$$
$$\beta = 1.2$$
$$Y_{cap} = 2.1\,\text{GPa}$$
$$P_{cap} = 1.72\,\text{GPa}$$



Damaged Glass

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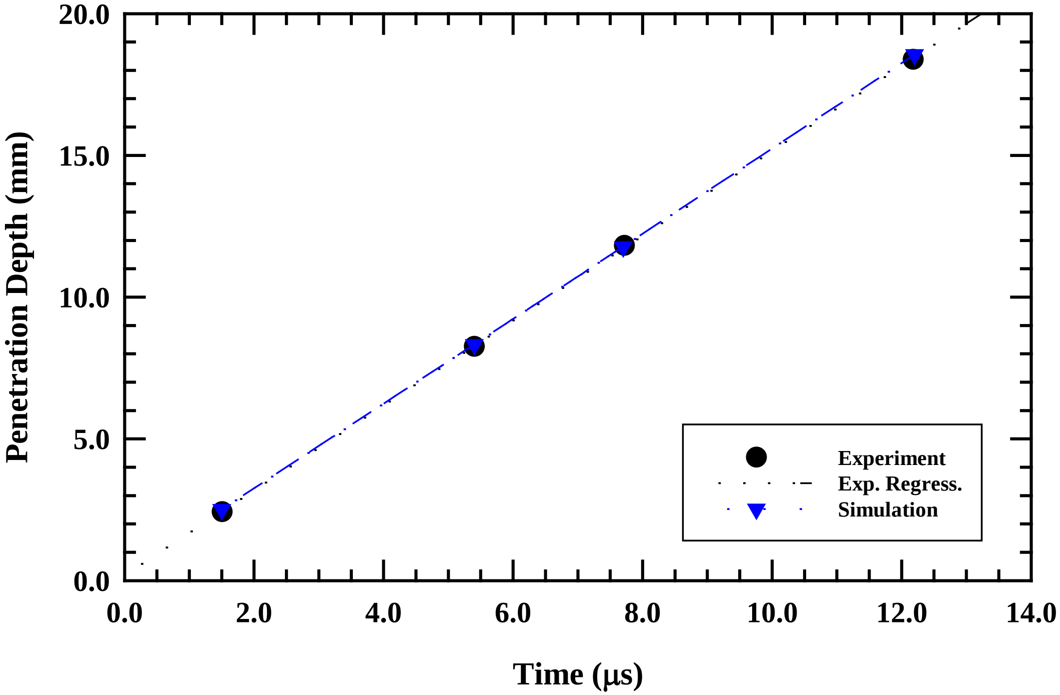
Analysis of Simulation Results

- *Analyzed the results of the numerical simulations like the experiments:*
  - *Determined the depth of penetration at the respective X-ray times*
  - *Conducted a linear regression fit on those simulated data points*
  - *Compared results to experimental data*

v_p = 2.238 km/s



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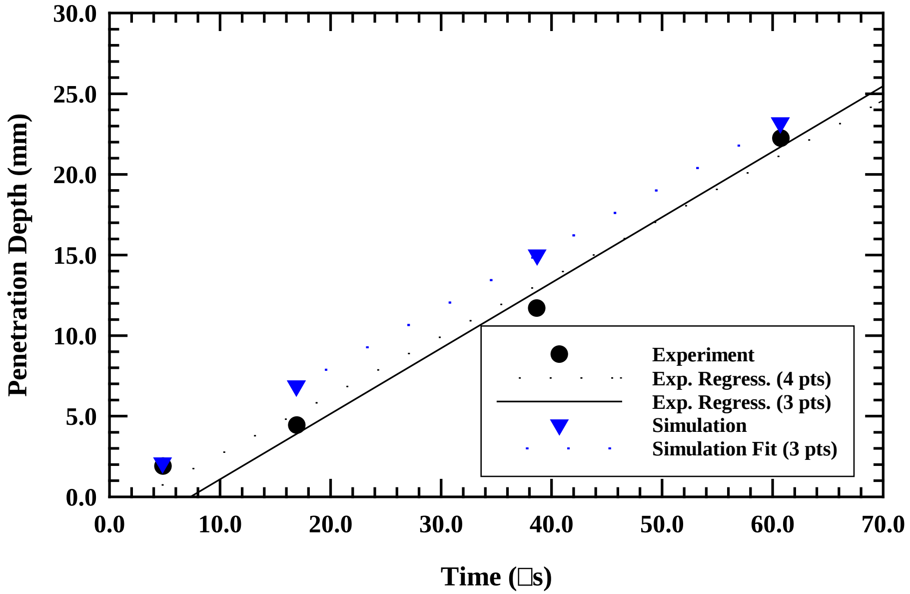
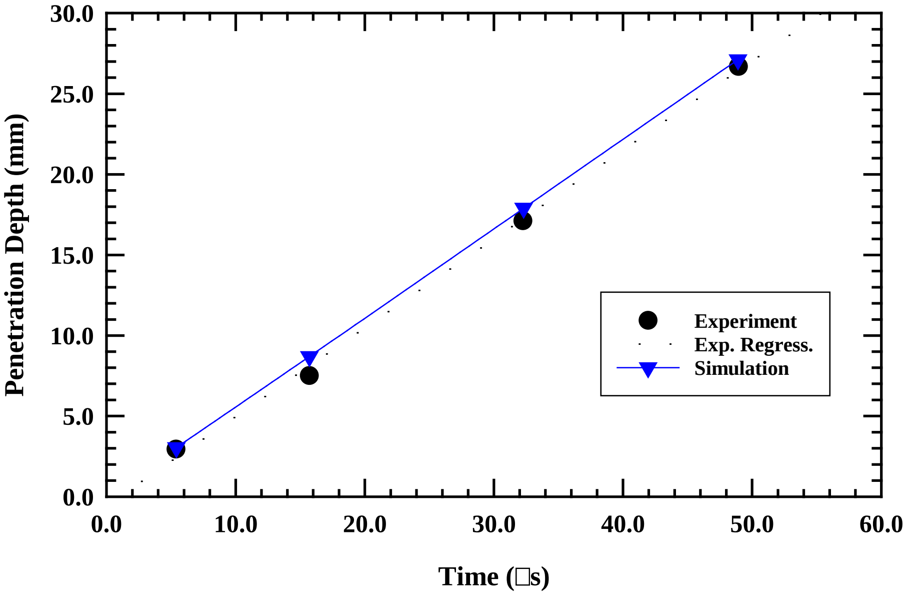
Penetration-Time Results





v_p = 0.768 km/s

v_p = 1.002 km/s

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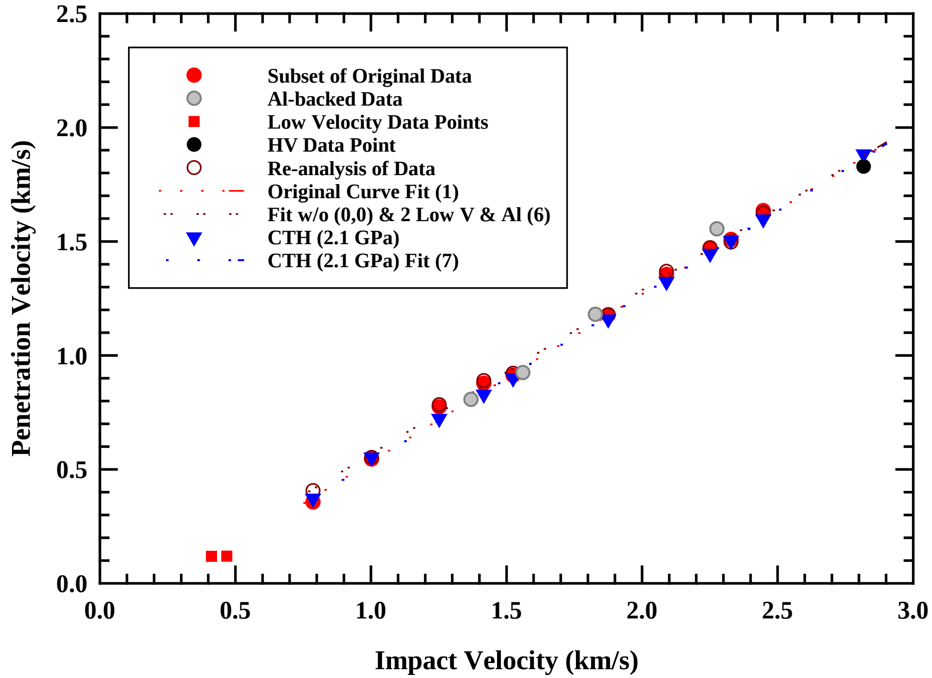
Analysis of All Experiments

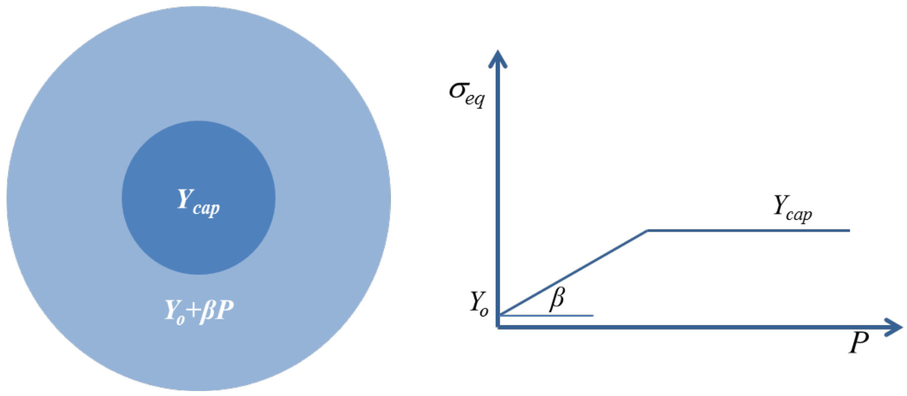


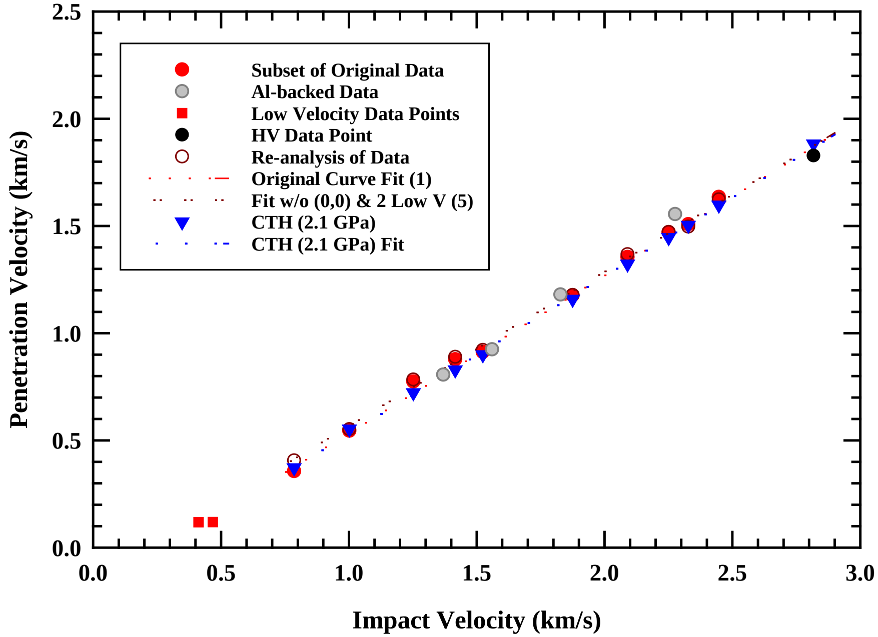
	Regression Fit	Fit Std. Error (km/s)	Slope Std. Error
Experiments	$u = 0.7200\text{v}_p - 0.1530$	0.0226	0.0129
Simulations	$u = 0.7289\text{v}_p - 0.1962$	0.0109	0.0062

Slopes differ only by 1.2%

Parametric Study

- There is some uncertainty in determination of the Drucker-Prager constitutive constants: ±10% on β and Y_{cap}





- Y_{cap} = 2.1 GPa → 1.89 GPa: tends to increase penetration at high v_p
- β = 1.2 → 1.1: tends to increase penetration at low v_p

Results of Parametric Study

Fit No.	Constitutive Constants	Regression Fit	Fit Std. Error (km/s)	Slope Std. Error
7	$\beta = 1.2$, $Y_{cap} = 2.1$ GPa	$u = 0.7289 v_p - 0.1962$	0.0109	0.0062
8	$\beta = 1.2$, $Y_{cap} = 1.89$ GPa	$u = 0.7427 v_p - 0.2102$	0.0130	0.0074
9	$\beta = 1.1$, $Y_{cap} = 2.10$ GPa	$u = 0.7214 v_p - 0.1745$	0.0107	0.0061

- *Decreased cap, slope increased 1.9%*
- *Decreased β, slope decreased by 1.0%*
- *If had decreased cap and β, slope would have tended to remain the same, but have slightly deeper penetration over the entire velocity range*

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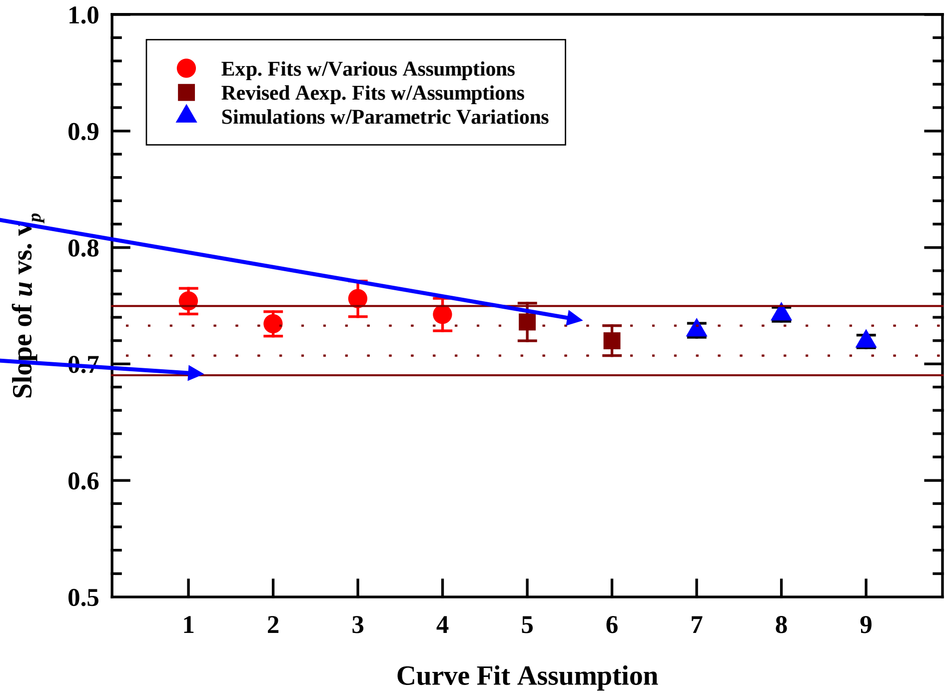
Comparison of Experiments and Simulations

- Compare slopes (penetration velocity as function of impact velocity)
  - Standard error in expt. slope
  - 95% confidence bound for the experimental slope
- The baseline simulation results fall within the uncertainty of the experimental results
- Might be tempted to state that Fit No. 9 is better than Fit 7
- Beware of numerology!



Summary & Conclusions

- Demonstrated that can reproduce reverse ballistics experiments of a gold rod into a borosilicate glass:
  - Using a Drucker-Prager constitutive model
  - Model constants determined from independent laboratory characterization experiments
  - Slight changes in the constitutive constants (representing the uncertainties from characterization) also reproduce the experimental data within experimental scatter
- Assumption of penetrating failed glass was validated
  - Provided not near the dwell-transition velocity where details of going from intact to damaged glass are important
- Glass is highly compressible, and important to have appropriate equation of state